Tapered Stripline Embedded in Inhomogeneous Media as Microwave Matching Line

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***Abstract*—A novel design method for a stripline microwave matching line is developed in this paper. Striplines considered have a tapered inner conductor embedded in an inhomogeneous dielectric material with continuous spatial variation of the relative permittivity. The employment of this kind of waveguide, as it is shown in this paper, ensures good matching properties in a wide frequency range. These matching properties can be controlled by means of two different factors: the taper of the stripline inner conductor and the relative permittivity spatial variation of the dielectric material filling the stripline. Starting from the nonuniform transmission-line theory, a novel closed analytical form for the input reflection coefficient of such lines is derived, and design formulas for the matching line are carried out. Finally, several applications that show the capability, flexibility, and fastness of the developed synthesis method are presented.**

***Index Terms*—Design formulas, matching line, stripline.**

I. Introduction

In MONOLITHIC-MICROWAVE integrated-circuit (MMIC) technology, microstrips and striplines are widely used because of several of their properties: large bandwidth, excellent miniaturization, small volume, small weight, etc. Moreover, the very easy passive circuits realization and the very good integration with chip devices make them very popular in microwave printed-circuit technology.

The stripline consists of an inner conductor embedded in a dielectric material that is sandwiched between two conducting planes. In the balanced form, the stripline has the inner conductor in the middle between the two ground planes (Fig. 1).

Since the principle mode of operation for such a waveguide is the $\mathrm{TEM}(\hat{x})$, it allows the field inside the stripline to be studied in a very fast and easy way. In fact, the TEM mode has a null cutoff frequency and, therefore, it can be studied as a static field.

Striplines have been widely studied starting from the end of the 1940s and the beginning of the 1950s [1]–[3]. In these early studies, the main tasks were the calculus of the characteristic impedance of the line and the derivation of a static capacitance model to completely represent the field in every transverse section of the lossless balanced stripline. This model takes into account both the reactive effect of the capacitance between the inner conductor and the two metallic shields and the fringing effect of the field at the boundary of the inner conductor. In Fig. 2, the capacitance model for the generic transverse section of the stripline is depicted as shown in [4].

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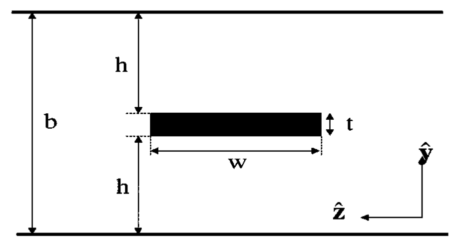

Fig. 1. Balanced stripline geometry.

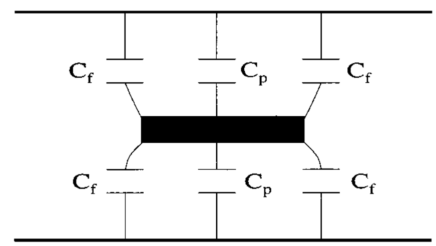

Fig. 2. Stripline capacitance model. C_p is the parallel-plate capacitance and C_f is the fringing capacitance.

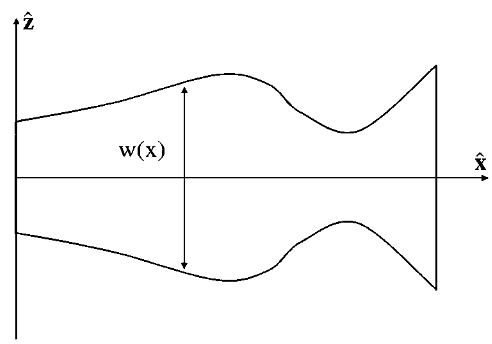

Fig. 3. Stripline taper.

In matching purposes, a wide frequency range in which reflections are very low (i.e., under a certain threshold) is often required. In order to achieve this goal, tapered lines can be successfully used [5]. Thus, if it has to match each other two uniform striplines with different characteristic impedances and a wide-band matching is required, a stripline with a tapered inner conductor (Fig. 3) can be used. Section by section the width of the inner conductor varies and, thus, the characteristic impedance of the line also varies in such a way that, at the two ends, it has the same values of the two uniform lines to be matched.

Recently, a new kind of matching method has been carried out by using inhomogeneous dielectric media [6], [7]. The dielectric material used has a continuous spatial variation of the relative permittivity and it also allows to perform a good matching on a large range of frequencies. These kind of materials can be obtained via nonuniform metallic inclusions with different shapes and dimensions in a host dielectric. Otherwise, typical examples of such materials are those used for optical waveguides and fibers with a spatial variation of the refractive index or semiconductor with a continuous doping profile.

However, the idea proposed in this paper is to merge the taper method and the use of inhomogeneous substrates in order to design very general matching lines with high performances and with multiple control capabilities. The striplines used have an inner conductor tapered along the $\hat{x}$-axis and the substrate with an arbitrary continuous permittivity profile along the same direction.

The introduced component is such that its matching properties can be controlled in two different ways: by using the taper of the inner conductor and/or by using the permittivity profile of the dielectric substrate. The aim of this paper is to find a model that yields to simple, accurate, and practical design formulas for these striplines used in microwave matching purposes.

II. Theory

A. Parameters of the Stripline

Let us consider the stripline capacitance model depicted in Fig. 2. The parallel-plate capacitance per unit length (in the absence of fringing) is given by

$$C_p = \epsilon_0\epsilon_r \frac{2w}{b-t} = 2\epsilon_0\epsilon_r \frac{\frac{w}{b}}{1-\frac{t}{b}}. \tag{1}$$

On the other hand, the fringing capacitance per unit length has been exactly computed by conformal mapping [8, p. 104, eq. 35] as follows:

$$C_f = \frac{\epsilon_0\epsilon_r}{\pi}\left\{\frac{2}{1-\frac{t}{b}}\log\left(1+\frac{1}{1-\frac{t}{b}}\right) - \left(\frac{1}{1-\frac{t}{b}}-1\right)\log\left[\frac{1}{\left(1-\frac{t}{b}\right)^2}-1\right]\right\}. \tag{2}$$

Referring to the capacitance model depicted in Fig. 2, the total capacitance per unit length of the stripline is given by

$$C_t = 2C_p + 4C_f.$$

Two of the most important parameters of any transmission line are its characteristic impedance (η) and phase factor (β). Assuming the TEM-mode propagation, these two parameters can be written as follows:

$$\eta = 120\frac{\pi\epsilon_0\epsilon_r}{\sqrt{\epsilon_r}C_t} \tag{3}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}. \tag{4}$$

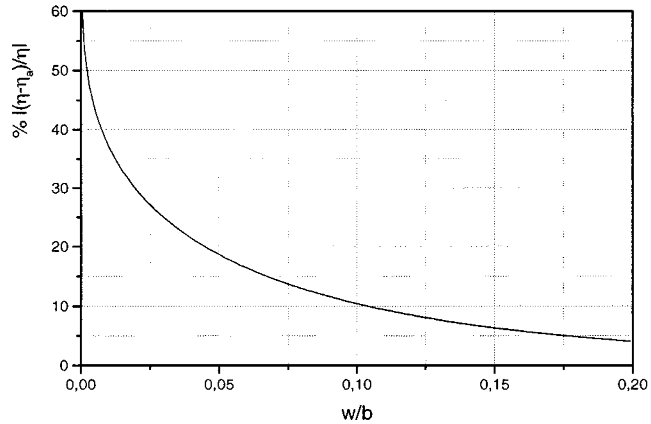

Fig. 4. Characteristic impedance relative error made using (6) instead of the exact formula [1], [10].

It is well known [1], [10] that if the thickness t of the inner conductor can be neglected, so that $t/b \ll 1$, in such a way that it can be imposed, $t = 0$, a closed analytical form for the characteristic impedance, based on conformal mapping, can be derived. This exact solution involves a ratio of two complete elliptic integrals of the first kind K and it is very simple, practical, and accurate. However, if the width (w) of the inner conductor is greater than $0.175b$, it will be demonstrated in the following that an approximated formula can be easily carried out for the characteristic impedance so that the relative error on the determination of the characteristic impedance is less than 5%.

By assuming $t = 0$ in (1) and in (2), the total capacitance per unit length is approximated by

$$C_t \simeq 4\epsilon_0\epsilon_r\left(\frac{w}{b}+\frac{2}{\pi}\log 2\right). \tag{5}$$

The characteristic impedance of the stripline in such a case is given by

$$\eta_a = \frac{30\pi}{\sqrt{\epsilon_r}}\frac{1}{\frac{w}{b}+\frac{2}{\pi}\log 2} \tag{6}$$

where the subscript a denotes that this is an approximated form. In Fig. 4, the relative error obtained using (6) instead of the exact formula is depicted as a function of the inner conductor normalized width (w/b).

As can be seen, the relative error on the characteristic impedance decreases as the inner conductor normalized width increases. Moreover, the approximated formula (6) leads to an error less than 5% if the normalized inner conductor width (w/b) is greater than 0.175. This makes (6) commonly used in analysis and synthesis practical problems regarding striplines.

In this paper, we use (6) and (4) for the characteristic impedance and the phase factor, respectively. These two formulas are effective section by section along the $\hat{x}$-direction and describe the propagation of the $\mathrm{TEM}(\hat{x})$ mode in an uniform stripline.

If we want to study nonuniform striplines that have an inner conductor tapered along the $\hat{x}$-axis, it can be shown that (6) and (4) are still effective [9]. This result comes from the fact that we can write them on each transverse section of the stripline.

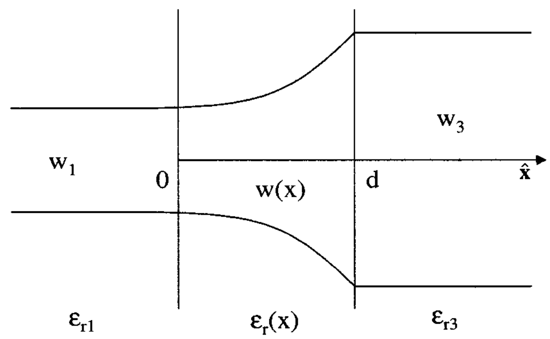

Fig. 5. Typical matching purpose.

For such a transmission line, we have, section by section, the same phase factor (β), but a different characteristic impedance. In fact, if the dielectric material that fills the stripline is homogeneous, (6) and (4) become

$$\eta(x) = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{1}{\dfrac{w(x)}{b} + \dfrac{2}{\pi}\log 2} \tag{7}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}. \tag{8}$$

Moreover, if the inner conductor has an uniform width and the dielectric material is inhomogeneous with a continuous variation of the relative permittivity along the $\hat{x}$-axis, (6) and (4) are also still effective section by section. In this case, we have either a variation of the characteristic impedance and a variation of the phase factor along the $\hat{x}$-direction.

Finally, if we use a nonuniform stripline filled by an inhomogeneous dielectric material, the characteristic impedance and phase factor can be written as follows:

$$\eta(x) = \frac{30\pi}{\sqrt{\epsilon_r(x)}} \frac{1}{\dfrac{w(x)}{b} + \dfrac{2}{\pi}\log 2} \tag{9}$$

$$\beta(x) = \omega\sqrt{\mu_0\epsilon_0\epsilon_r(x)}. \tag{10}$$

B. Matching Lines

Let us consider now the use of a tapered stripline filled by an inhomogeneous material as a matching line. A typical matching purpose is depicted in Fig. 5.

It has to match two striplines with different characteristic impedance (η_1 and η_3, respectively). Let us suppose, as a general case, that these two lines have different inner conductor widths and that they are embedded in different dielectric materials (it is clear that the cases in which $w_1 = w_3$ or $\epsilon_{r1} = \epsilon_{r3}$ are sub-cases).

A $\hat{x}$ tapered stripline with its inner conductor embedded in a dielectric material whose relative permittivity varies along the $\hat{x}$-direction is employed to solve this kind of matching problem. The choice of this tapered inhomogeneous matching line depends on the fact that a good matching can be performed for a very large frequency range [5].

This is not the only possible choice to make up a matching line. For instance, a classical $\lambda/4$ transformer can be used, but the results are very poor. We have a good matching, in fact, only for narrow frequency bands centered around the odd multiple of the fundamental frequency (i.e., the frequency for which the transformer is designed). As can be seen from Fig. 6, the matching frequency band can be improved by using a cascade of $\lambda/4$ transformers.

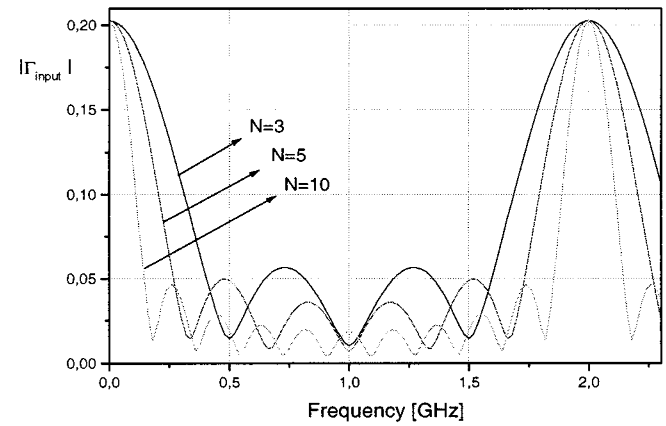

Fig. 6. Magnitude of the input reflection coefficient as a function of frequency (in gigahertz) by using a cascade of $N\lambda/4$ transformers. (The two uniform striplines to be matched have $\eta_1 = 75\,\Omega$ and $\eta_3 = 50\,\Omega$, respectively. The free-space fundamental frequency is $f_0 = 1$ GHz.)

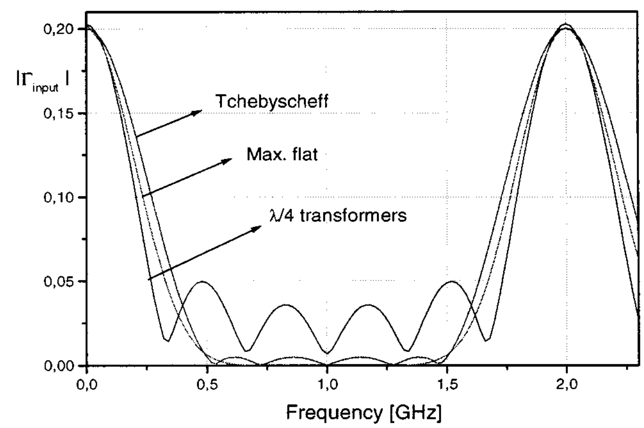

Fig. 7. Magnitude of the input reflection coefficient as a function of frequency (in gigahertz) by using a cascade of $N = 5$ $\lambda/4$, maximally flat and Chebyshev transformers, (The two uniform striplines to be matched have $\eta_1 = 75\,\Omega$ and $\eta_3 = 50\,\Omega$, respectively. The free-space fundamental frequency is $f_0 = 1$ GHz.)

It is well known that a better bandwidth can be achieved by using maximally flat or Chebyshev transformers, as shown in Fig. 7.

However, these matching lines, based on a cascade of uniform transformers, have several disadvantages. The abrupt transition regions between adjacent sections, in fact, exhibits reactive discontinuities, which introduce reflections and, thus, degrade the matching performance at the higher microwave frequencies. On the other hand, the length of the matching line increases with the number of the used transformers and, finally, if wider matching bandwidths are needed, it has to find a way to knock down the peaks of the reflection-coefficient magnitude for the even multiples of the fundamental frequency.

C. Reflection Coefficient

If we want to perform the matching by using a tapered stripline with the inner conductor embedded in an inhomogeneous material, we have to compute the input reflection coefficient for a nonuniform transmission line. It is well known that the reflection coefficient for this kind of lossless line satisfies the following nonlinear Riccati equation [11]:

$$\frac{d\Gamma(x)}{dx} - 2j\beta(x)\Gamma(x) + \frac{1-\Gamma^2(x)}{2}\,\frac{d\log\eta(x)}{dx} = 0. \quad (11)$$

In matching purposes, section by section, the reflection coefficient along the nonuniform line is very small so that it can be imposed, i.e., : $|\Gamma(x)|^2 \ll 1$. In [12], it is shown that via this assumption, the Riccati equation becomes an ordinary linear equation and that, once it is solved, the input reflection coefficient for an L-long nonuniform transmission line is

$$\Gamma_{\text{input}} = \int_0^L \frac{1}{2}\,\frac{d\log\eta(x)}{dx}\,\exp\left(-2j\int_0^x \beta(x')\,dx'\right)\,dx. \quad (12)$$

In the general case, if the functional profiles of the characteristic impedance and phase factor are arbitrary, (12) does not yield a closed form because it involves two numerical integrations.

Therefore, even if (12) can be successfully used in a matching line analysis purpose, it is not suitable for design purposes. In this kind of problems, in fact, once the parameters of the two striplines to be matched are given, the length L of the matching line, the taper $w(x)$ of the inner conductor, and the relative permittivity profile $\epsilon_r(x)$ have to be computed. In the general case, (12) does not yield a closed analytical form, thus, we cannot straightforwardly derive the design parameters among which, for instance, the length L of the matching line.

In the Section II-D, the generalized nonuniform transmission-line theory is exposed via which a closed analytical form for the reflection coefficient is carried out (Section II-E).

D. Generalized Nonuniform Transmission-Line Theory

The coupled linear equations for a nonuniform transmission line can be written as follows:

$$\begin{cases} \dfrac{dV(x)}{dx} = -Z(x)I(x) \\ \dfrac{dI(x)}{dx} = -Y(x)V(x) \end{cases} \quad (13)$$

where $Z(x) = Z_0 f(x)$ and $Y(x) = Y_0 g(x)$ are the impedance and admittance of the line, respectively. In the general case, $f(x)$ and $g(x)$ are arbitrary functions. Z_0 and Y_0 are, instead, constants that are imaginary for a lossless line.

The second-order differential voltage equation is

$$\frac{d^2V(x)}{dx^2} - \frac{1}{f(x)}\,\frac{df(x)}{dx}\,\frac{dV(x)}{dx} - Z_0Y_0 f(x)g(x)V(x) = 0. \quad (14)$$

Although this equation does not have a closed analytical solution in the general case, for several specific impedance and admittance profiles, a closed form can be found out (i.e., exponential line, linearly tapered line, hypergeometric line, hyperbolic line, etc.).

In [13] and [14], a generalization of the nonuniform transmission line, whose solution can be expressed in a closed analytical form, has been proposed. Via the variable substitution $x \Rightarrow u(x)$, where $u(x)$ is an arbitrary, derivable, and not null derivative function, (14) becomes

$$\frac{d^2V(x)}{du^2} - \frac{1}{f(x)}\,\frac{d}{dx}\left[\frac{f(x)}{du/dx}\right]\frac{dV(x)}{du} - Z_0Y_0\frac{f(x)g(x)}{(du/dx)^2}\,V(x) = 0. \quad (15)$$

This equation is the second-order differential voltage equation for a *generalized* nonuniform transmission line.

To clarify the concept of generalization, let us consider as an example an exponential transmission line. Impedance and admittance profiles are of the form

$$\begin{cases} Z(x) = Z_0 e^{qx} \\ Y(x) = Y_0 e^{-qx} \end{cases} \quad (16)$$

where q is the taper factor of the line. Inserting these profiles into (14), it becomes

$$\frac{d^2V(x)}{dx^2} - q\frac{dV(x)}{dx} - Z_0Y_0V(x) = 0. \quad (17)$$

The solution of this equation is given by

$$V(x) = c_1 \exp\left[\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + Z_0Y_0}\right] x + c_2 \exp\left[\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + Z_0Y_0}\right] x. \quad (18)$$

Let us now consider an arbitrary function $u(x)$. If the impedance and admittance profiles of a nonuniform transmission line are of the following form:

$$\begin{cases} Z(x) = Z_0\dfrac{du(x)}{dx}\,e^{qu(x)} \\ Y(x) = Y_0\dfrac{du(x)}{dx}\,e^{-qu(x)} \end{cases} \quad (19)$$

(15) becomes the same kind as (17) and its solution is

$$V(x) = c_1 \exp\left[\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + Z_0Y_0}\right] u(x) + c_2 \exp\left[\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + Z_0Y_0}\right] u(x). \quad (20)$$

Equation (19) defines the *generalized* exponential line (it is clear, in fact, that if $u(x) = x$ (19) and (20) become (16) and (18), respectively).

E. Closed Form for the Reflection Coefficient

Although the generalization of the nonuniform transmission-line theory can be applied to all the lines for which a closed-form voltage solution can be derived, here we only consider the generalized exponential one. Using this kind of line, a very simple closed analytical form for the reflection coefficient can be carried out.

In the lossless case, the characteristic impedance and phase factor for such a line are

$$\begin{cases} \eta(x) = \eta_0 e^{qu(x)} \\ \beta(x) = \beta_0 \dfrac{du(x)}{dx} \end{cases} \tag{21}$$

where $\eta_0 = \sqrt{Z_0/Y_0}$ and $j\beta_0 = \sqrt{Z_0 Y_0}$.

Let us now consider the matching problem depicted in Fig. 5. Inserting (21) in (12), the following closed analytical form for the input reflection coefficient is obtained:

$$\Gamma_{\text{input}} = Ae^{-j\beta_0[u(L)-u(0)]}\text{sinc}\Big\{\beta_0\big[u(L)-u(0)\big]\Big\} \tag{22}$$

where

$$A = \log\left(\sqrt{\frac{\eta_3}{\eta_1}}\right) \quad \text{sinc}(x) = \frac{\sin(x)}{x}$$

and η_3 and η_1 are the characteristic impedances of the two uniform lines to be matched.

This analytical closed form for the reflection coefficient can be successfully used both in analysis and synthesis matching-line purposes. In [7], it is shown that analysis problems using (22) instead of (12) lead to a strongly decrease of computation time because the two numerical integrations disappear. Synthesis examples are also proposed in [7] and they show how design formulas for the matching line can be directly derived.

F. Synthesis of the Matching Line

Referring to the Fig. 5 and to the formula (22), we have to find the unknown function $u(x)$ and the relationship between the relative permittivity profile and the taper of the inner conductor such that the closed form for the reflection coefficient can be successfully used. So, by comparing the second of (21) with (10), the arbitrary function $u(x)$ can be expressed in term of the relative permittivity profile of the dielectric material

$$u(x) = \int \sqrt{\epsilon_r(x)}\, dx. \tag{23}$$

On the other hand, by comparing the first of (21) with (9), the relationship between the permittivity profile $\epsilon_r(x)$ and conductor taper $w(x)$ can be derived as follows:

$$\frac{w(x)}{b} = -\frac{2}{\pi}\log 2 + C\frac{\exp\left(q\int\sqrt{\epsilon_r(x)}\,dx\right)}{\sqrt{\epsilon_r(x)}} \tag{24}$$

where C is an integration constant.

In design problems of the kind shown in Fig. 5, the typical input parameters (i.e., known parameters) are the geometrical and electromagnetic characteristics of the two striplines to be matched, i.e., w_1, w_3, ϵ_{r1}, and ϵ_{r3}. Another important input parameter is the lower frequency (f_0) for which a good matching has to be performed.

Instead, the output parameter of the matching line are the length L of the nonuniform stripline, the dielectric material permittivity profile $\epsilon_r(x)$, and the inner conductor taper profile $w(x)$.

Let us now consider that the dielectric material is not a dispersive material. The overall equations we can write are as follows:

$$\begin{cases} \epsilon_r(x=0) = \epsilon_{r1} \\ \epsilon_r(x=L) = \epsilon_{r3} \\ w(x=0) = w_1 \\ w(x=L) = w_3 \\ 2\pi f_0\sqrt{\mu_0\epsilon_0}|u(L)-u(0)| = n\pi \end{cases} \tag{25}$$

where n is a positive integer starting from one. Some explanations about these equations have to be given. The first four are derived from the boundary conditions at the ends of the matching line and it has to be remarked that, by means of (24), they are not all independent. Thus, only three of them can be used in the design of the matching line. Instead, the last equation is obtained by imposing that the sinc argument in (22) is an integer multiple of π at the f_0 frequency where the choice of n depends on the required quality of the matching we have to perform. The explanation of this statement is in the following. It is well known that the voltage standing-wave ratio (VSWR) is related to the magnitude of the reflection coefficient via

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

thus, once $\text{VSWR}^{\max}$ is given, the maximum allowed magnitude for the reflection coefficient is

$$|\Gamma|^{\max} = \frac{\text{VSWR}^{\max}-1}{\text{VSWR}^{\max}+1}. \tag{26}$$

This value has to be compared with the magnitude of Γ_{input} and, starting from f_0, Γ_{input} has to be less than it. Since Γ_{input} exhibits a sinc behavior, $|\Gamma|^{\max}$ has to be compared with the amplitude of the sinc sidelobes. Therefore, the right-hand-side value of n can be derived as the smallest positive integer that satisfies the following inequality:

$$\frac{\text{VSWR}^{\max}-1}{\text{VSWR}^{\max}+1} > \left|A\text{sinc}\left[(2n+1)\frac{\pi}{2}\right]\right|. \tag{27}$$

Thus, the integer n is associated with the nth sidelobe of the sinc and, if $|\Gamma|^{\max}$ is greater than the amplitude of this sidelobe, it is also greater than all the following ones. In addition, since we are sure that (27) also holds at the nth zero position, in order to extend the good matching frequency range, we can calculate the proper length of the matching line from the fifth equation of (25).

Once the proper value of n has been determined, the designer can arbitrarily choose either the kind of function describing the taper of the inner conductor or the permittivity profile

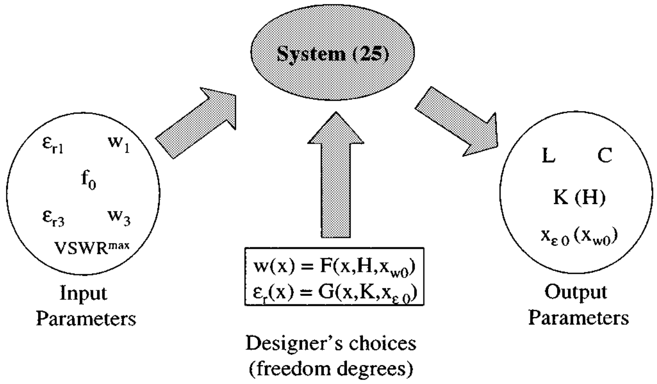

Fig. 8. Design method.

of the dielectric material. The associate permittivity profile or taper profile is then given by (24). In order to satisfy the two boundary conditions at the ends of the matching line, both $w(x)$ and $\epsilon_r(x)$ are chosen with two freedom degrees (an amplitude and a shift coefficient): $w(x) = F(x, H, x_{w0})$ and $\epsilon_r(x) = G(x, K, x_{\epsilon 0})$. Thus, if the designer arbitrarily chooses $w(x)$ [and derives $\epsilon_r(x)$ via (24)], he has to determine the length of the matching line L, H, x_{w0}, and the integration constant C in (24), but, if he arbitrarily chooses $\epsilon_r(x)$ [and derives $w(x)$ via (24)], he has to determine L, K, $x_{\epsilon 0}$, and C.

The design method developed in this section for the matching line is discussed further in Fig. 8.

III. Numerical Results

In this section, we show several applications of the design method presented above. First of all, let us consider a matching problem between two uniform, lossless, matched, and balanced striplines (referred to as striplines 1 and 3) with different normalized widths of the inner conductor and with different dielectric materials. Stripline 1 has an inner conductor, whose normalized width is $w_1/b = 0.4$, and a dielectric material with relative permittivity $\epsilon_{r1} = 2.33$ (RT/Duroid). The other stripline (stripline 3) is instead characterized by the following parameters: $w_3/b = 1.2$ and $\epsilon_{r3} = 6.80$ (beryllium oxide). The maximum allowed VSWR is 1.5 and the matching has to be effective starting from the frequency $f_0 = 3$ GHz.

Now that the six design specifications (i.e., input parameters) are given, the designer can arbitrarily choose either the law of relative permittivity spatial variation along the $\hat{x}$-axis, provided that the taper profile is given through (24), or the taper profile, provided that the variation of relative permittivity is again obtained via (24). Let us choose, for instance, a squared spatial variation for the relative permittivity along the x-axis, i.e., $\epsilon_r(x) = H(x + x_0)^2$. The inequality (27) is satisfied for $n = 1$ and the solution of the system (25) gives the following output parameters: $L = 2.418$ cm, $H \simeq 2000$, $x_0 = 0.03413$, and $C = 0.0153$.

The output parameters computed via the application of the novel method developed in the previous section allow the designer to construct the proper matching line. In Fig. 9, the overall normalized width of the matching line inner conductor as a function of x is depicted.

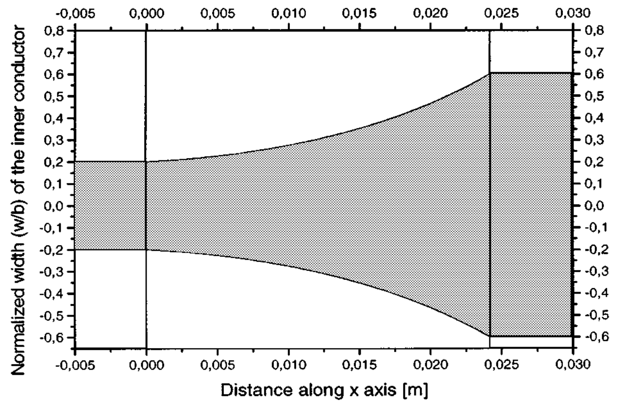

Fig. 9. Normalized inner conductor width as a function of x. The input parameters are $w_1/b = 0.4$, $w_3/b = 1.2$, $\epsilon_{r1} = 2.33$, $\epsilon_{r3} = 6.80$, $f_0 = 3$ GHz, and $\mathrm{VSWR}^{\max} = 1.5$.

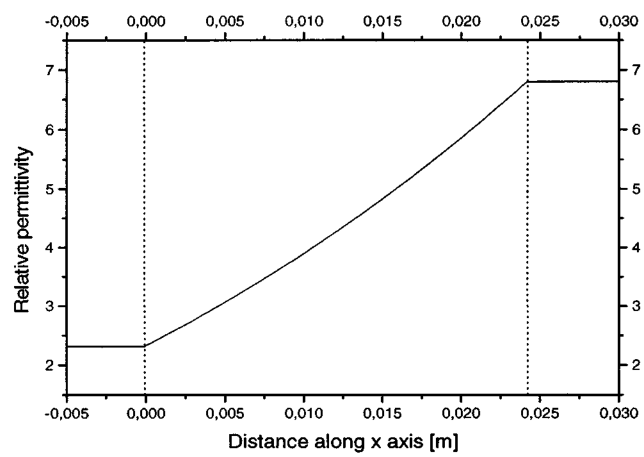

Fig. 10. Relative permittivity of the matching line as a function of x. The input parameters are $w_1/b = 0.4$, $w_3/b = 1.2$, $\epsilon_{r1} = 2.33$, $\epsilon_{r3} = 6.80$, $f_0 = 3$ GHz, and $\mathrm{VSWR}^{\max} = 1.5$.

Instead, in Fig. 10, the spatial variation of the dielectric relative permittivity along the $\hat{x}$-axis is shown.

Moreover, the numerical result obtained for the length of the matching line show that the use of tapered striplines with inhomogeneous substrates instead of a cascade of uniform transformers (i.e., $\lambda/4$, maximally flat or Chebyshev) allows the construction of a shorter matching line. This yields several important advantages in order to make these components very compact.

Finally, in Fig. 11, the behavior of the VSWR at the input section of the matching line is reported as a function of frequency.

It is worth noticing that the design specification about the $\mathrm{VSWR}^{\max}$ can be well satisfied by means of the high-pass frequency behavior of such a matching line because we consider a non-frequency-dispersive dielectric material. In a frequency-dispersive case, in fact, we should have a frequency behavior different from a $|\mathrm{sinc}|$ profile for the magnitude of the reflection coefficient and, thus, a different plot for the VSWR at the input section of the matching line. However, it is clear

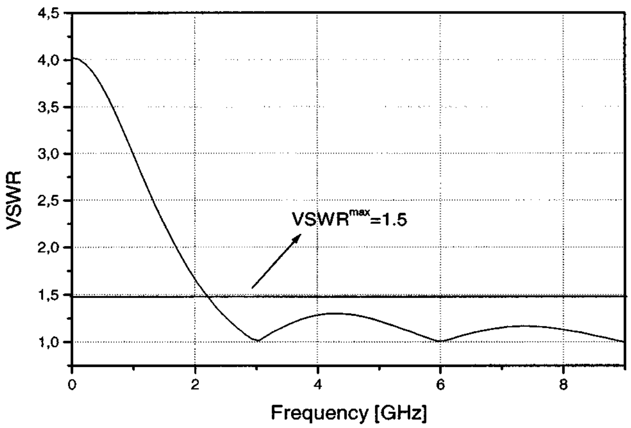

Fig. 11. VSWR at the input section of the matching line as a function of frequency. The input parameters are $w_1/b = 0.4$, $w_3/b = 1.2$, $\epsilon_{r1} = 2.33$, $\epsilon_{r3} = 6.80$, $f_0 = 3$ GHz, and $\mathrm{VSWR}^{\max} = 1.5$.

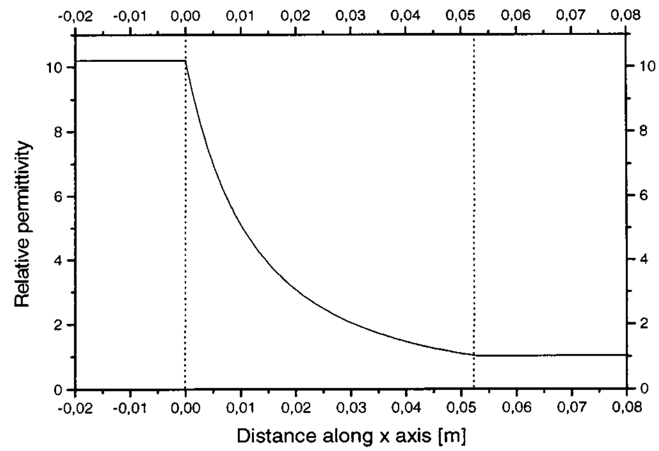

Fig. 12. Relative permittivity of the matching line as a function of x. The input parameters are: $w_1/b = 0.5$, $w_3/b = 0.5$, $\epsilon_{r1} = 1$, $\epsilon_{r3} = 10.2$, $f_0 = 5$ GHz, and $\mathrm{VSWR}^{\max} = 1.15$.

that the input reflection coefficient closed form (22) is also effective if we have to consider the frequency dispersion of the dielectric material and, thus, also the relative permittivity as a function of frequency. In fact, if $\epsilon_r = \epsilon_r(x, \omega)$, (23) becomes $u(x, \omega) = \int \sqrt{\epsilon_r(x, \omega)}\, dx$ and, in (22), $u(L)$ and $u(0)$ become $u(L, \omega)$ and $u(0, \omega)$. No other changes are needed. This means that the argument of the sinc in (22) is no longer a linear function of the frequency and, thus, the last equation of the system (25) is no longer effective. The needed changes to this equation will be the subject of future study.

Referring to Fig. 11, it can be seen that, although a perfect matching (i.e., $\mathrm{VSWR} = 1$) is achieved for only specific frequency values, starting from f_0, the $\mathrm{VSWR}^{\max}$ specification is fully satisfied. The theory of nonuniform transmission lines ensures that a complete matching for all the frequency band can be performed only if the matching line has an infinite length. Thus, the closer $\mathrm{VSWR}^{\max}$ is to one, the bigger the length of the matching line. For instance, referring to the previous matching case, if the $\mathrm{VSWR}^{\max}$ drops down from 1.5 to 1.1, the length of the line arises from 2.418 to 9.671 cm.

In the following, it is shown that the synthesis method for the matching line developed in the previous section is also effective in physical situations less general than that shown in the first numerical example. Let us consider here two uniform lossless matched balanced striplines (striplines 1 and 3, respectively), which have the same normalized width of the inner conductor $w_1/b = w_3/b = 0.5$. Stripline 1 is filled with air ($\epsilon_{r1} = 1$), stripline 3 with alumina ($\epsilon_{r3} = 10.2$), the maximum for the VSWR is $\mathrm{VSWR}^{\max} = 1.15$ and the lower matching frequency is $f_0 = 5$ GHz. The most straightforward way to design a matching line in this case is to choose a stripline with an uniform inner conductor ($w/b = 0.5$) and with a dielectric material whose relative permittivity continuously varies from 1 to 10.2. Assuming a constant normalized width w/b for the inner conductor of the matching line, (24) can be solved for the permittivity profile of the dielectric material and a parabolic law of the kind $\epsilon_r(x) = H(x + x_0)^2$ can be discovered. In this case (27), inequality is satisfied for $n = 3$; the length of the matching line is $L = 5.321$ cm, the integration constant in (24) is $C = 0.9413$, and the two permittivity parameters are $H \simeq 0.0600$ and $x_0 = 0.0243$.

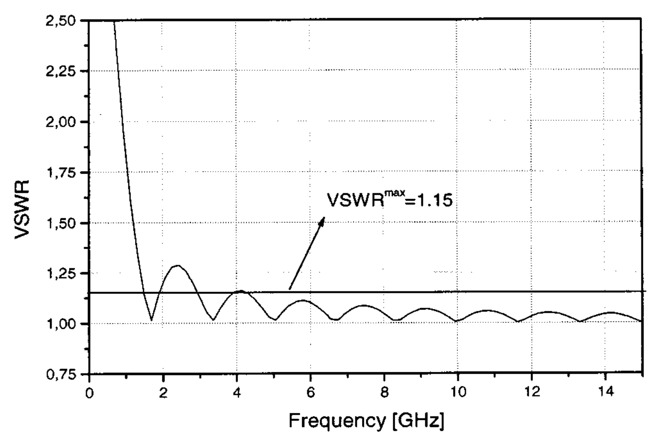

Fig. 13. VSWR at the input section of the matching line as a function of frequency. The input parameters are: $w_1/b = 0.5$, $w_3/b = 0.5$, $\epsilon_{r1} = 1$, $\epsilon_{r3} = 10.2$, $f_0 = 5$ GHz, and $\mathrm{VSWR}^{\max} = 1.15$.

In Fig. 12, the relative permittivity profile of the dielectric material filling the matching line is shown as a function of x.

In Fig. 13, the plot of the VSWR at the input section of the matching line is reported. As can be seen, also in this case, the given matching specification ($\mathrm{VSWR} < 1.15$) is fully satisfied.

The last numerical example that is proposed regards another special case of the general theory developed in the previous section. Let us consider two uniform lossless matched, balanced striplines, i.e., striplines 1 and 3. They have the same substrate (RT/Duroid, $\epsilon_{r1} = \epsilon_{r3} = 2.33$), but different normalized widths of the inner conductor ($w_1/b = 0.5$ and $w_3/b = 2.2$). The maximum VSWR allowed on the input section of the matching line is $\mathrm{VSWR}^{\max} = 1.2$ and the lower matching frequency is $f_0 = 7$ GHz. In this case, we have to design a matching line with the same dielectric material of the other two ($\epsilon_r = 2.33$) and with an inner conductor whose normalized width varies from $w/b = 0.5$ to $w/b = 2.2$. Assuming a constant value for ϵ_r

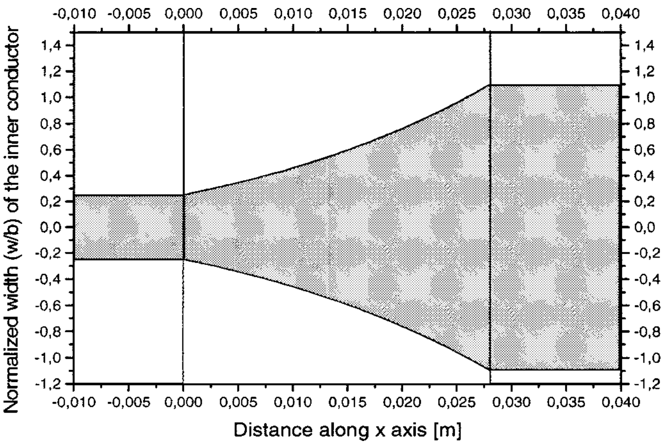

Fig. 14. Normalized inner conductor width as a function of x. The input parameters are: $w_1/b = 0.5$, $w_3/b = 2.2$, $\epsilon_{r1} = 2.33$, $\epsilon_{r3} = 2.33$, $f_0 = 7$ GHz, and $\text{VSWR}^{\max} = 1.2$.

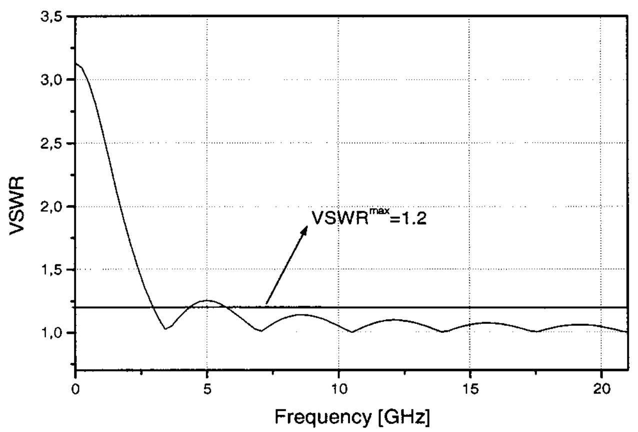

Fig. 15. VSWR at the input section of the matching line as a function of frequency. The input parameters are: $w_1/b = 0.5$, $w_3/b = 2.2$, $\epsilon_{r1} = 2.33$, $\epsilon_{r3} = 2.33$, $f_0 = 7$ GHz, and $\text{VSWR}^{\max} = 1.2$.

in (24), the taper of the inner conductor has an exponential law $-2/\pi \log 2 + Ke^{x/x_0}$. Inequality (27) gives $n = 2$ and, via the solution of the system (25), the following parameters are obtained: $L = 2.806$ cm, $K = 0.9413$, $x_0 = 0.0272$, and $C = 1.4300$. In Fig. 14, the overall normalized width of the central conductor as a function of the distance along the $\hat{x}$-axis is reported.

In Fig. 15, the VSWR on the input section of the matching line as a function of frequency is depicted. As can be seen, starting from $f_0 = 7$ GHz, the matching fully satisfies the design specifications.

IV. Conclusions

In this paper, a new type of wide-band matching line has been proposed. This matching line consists of a stripline with a tapered inner conductor embedded in an inhomogeneous dielectric material whose relative permittivity varies continuously along the energy propagation direction.

Using the nonuniform transmission-line theory, a closed analytical form for the input reflection coefficient of the matching line has been carried out. On the base of this formula, a complete, accurate, and very fast design method for this kind of matching line has been developed.

Finally, several numerical results have been presented to show how the novel design method can be successfully used in practical situations. First, the very general matching problem in which it has to match two other striplines with different inner conductor widths and different dielectric substrates has been considered and solved via the novel method developed here. The particular matching problems in which the two striplines to be matched then have different inner conductor widths, but the same dielectric substrate, and vice versa, have been solved as sub-cases.

The main attractive of the novel method developed in this paper is in the capability of deriving in a straightforward manner the length of the matching line, relative permittivity profile, and taper of the inner conductor. The main physical result of the new kind of matching line proposed instead is the capability of obtaining a good matching on a wide frequency range by means of two control keys: the conductor taper and dielectric inhomogeneity.

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